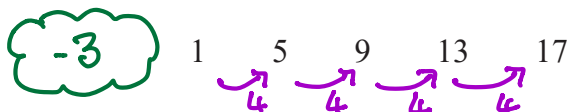


Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 Here are the first five terms of an arithmetic sequence.



- (a) Find an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

$$4n - 3$$

$$\underline{4n - 3}$$

(2)

The  $n$ th term of another arithmetic sequence is  $3n + 5$

- (b) Find an expression, in terms of  $m$ , for the  $(2m)$ th term of this sequence.

$$\begin{aligned} 3(2m) + 5 \\ = 6m + 5 \end{aligned}$$

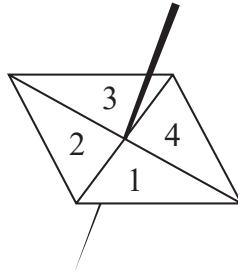
$$\underline{6m + 5}$$

(1)

(Total for Question 1 is 3 marks)



2 Here is a biased 4-sided spinner.



The table gives the probabilities that, when the spinner is spun once, it will land on 1 or it will land on 3

<b>Number</b>	1	2	3	4
<b>Probability</b>	0.26	0.28	0.18	0.28

The probability that the spinner will land on 2 is equal to the probability that the spinner will land on 4

Ravina is going to spin the spinner a number of times.

Ravina works out that an estimate for the number of times the spinner will land on 3 is 45

Work out an estimate for the number of times the spinner will land on 4

Probability of 2 & 4

$$1 - (0.26 + 0.18) = 1 - 0.44$$

$$= 0.56$$

$$\text{then } 0.56 \div 2 = 0.28$$

Using  $P(3) = 0.18$

$$\begin{array}{l} \div 9 \downarrow 0.18 = 45 \text{ times} \\ \times 14 \downarrow 0.02 = 5 \text{ times} \\ \phantom{\times 14 \downarrow} 0.28 = 70 \end{array}$$

70

(Total for Question 2 is 4 marks)



- 3 (a) Find the highest common factor (HCF) of 56 and 84  
Show your working clearly.

$$56 = 7 \times 2^3$$

$$84 = 7 \times 3 \times 2^2$$

$$\text{HCF} = 2^2 \times 7$$

$$= 4 \times 7$$

$$= 28$$

.....  
(2)

- (b) Find the lowest common multiple (LCM) of 60 and 72  
Show your working clearly.

$$60 = 2^2 \times 3 \times 5$$

$$72 = 8 \times 9 \\ = 2^3 \times 3^2$$

$$\text{HCF} = 2^2 \times 3$$

$$= 12$$

$$\text{LCM} = 12 \times 5 \times 2 \times 3$$

$$= 60 \times 2 \times 3$$

$$= 120 \times 3$$

$$= 360$$

.....  
360

(2)

(Total for Question 3 is 4 marks)



- 4 The diagram shows parts of three regular polygons, **A**, **B** and **C**, meeting at a point.

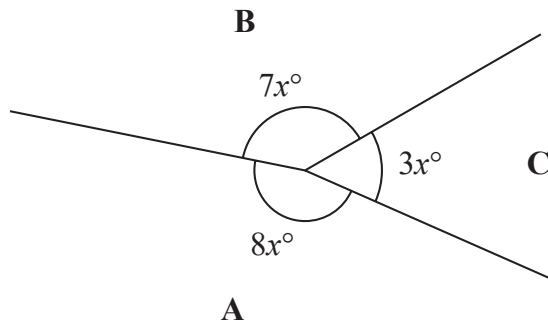


Diagram NOT accurately drawn

Polygon **B** has  $n$  sides.

Work out the value of  $n$ .

$$\begin{aligned}
 7x + 8x + 3x &= 360 \\
 18x &= 360 \\
 x &= \frac{360}{18} = 20
 \end{aligned}$$

$$\begin{aligned}
 \text{interior angle of B} &= 7x \\
 &= 7 \times 20 = 140
 \end{aligned}$$

$$\begin{aligned}
 \text{so exterior angle} &= 180 - 140 \\
 &= 40
 \end{aligned}$$

$$\begin{aligned}
 \text{Number of sides} &= \frac{360}{40} \\
 &= 9
 \end{aligned}$$

$$n = \dots\dots\dots 9 \dots\dots\dots$$

(Total for Question 4 is 4 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



5 (a) Expand and simplify  $(n - 6)(n + 4)$

$$n \times n = n^2$$

$$n \times 4 = 4n$$

$$-6 \times n = -6n$$

$$-6 \times 4 = -24$$

$$n^2 + 4n - 6n - 24$$

$$= n^2 - 2n - 24$$

$$\frac{n^2 - 2n - 24}{(2)}$$

(b) Solve  $2x - 3 = \frac{3x - 5}{4}$

Show clear algebraic working.

$$4(2x - 3) = \frac{3x - 5}{4} \times 4$$

$$\begin{array}{r} 8x - 12 = 3x - 5 \\ -3x \end{array}$$

$$\begin{array}{r} 5x - 12 = -5 \\ +12 \end{array}$$

$$5x = 7$$

$$x = \frac{7}{5} = 1\frac{2}{5}$$

$$x = \frac{7}{5} \text{ (or } 1\frac{2}{5} \text{ or } 1.4)$$

(3)

(Total for Question 5 is 5 marks)



6 Asha bought an apartment.

The table gives information about the value of apartments, in euros, and the annual service charge band.

Value ( $x$ euros)	Service charge band
$x \geq 700\,000$	A
$600\,000 \leq x < 700\,000$	B
$500\,000 \leq x < 600\,000$	C
$400\,000 \leq x < 500\,000$	D
$0 < x < 400\,000$	E

In 2021, the value of Asha's apartment was 634 400 euros.

The value of Asha's apartment had increased by 4% from its value in 2020

- (a) Has the annual service charge band changed for Asha's apartment?  
Show your working clearly.

2021 634 400  $\rightarrow$  Band B

$$\begin{aligned} \underline{2020} \quad 634\,400 &= 104\% \\ &= 1\% \quad \downarrow \div 104 \\ 610\,000 &= 100\% \quad \downarrow \times 100 \end{aligned}$$

610 000  $\rightarrow$  Band B so no change (3)

Pam bought a boat.

In each year after Pam bought the boat, the value of the boat depreciated by 15%

- (b) Work out the total percentage by which the value of the boat had depreciated by the end of the second year after Pam bought the boat.

$$100 - 15 = 85 \quad 85\% = 0.85$$

$$100 \times 0.85 \times 0.85 = 72.25$$

$$100 - 72.25 = 27.75$$

27.75 %

(3)

(Total for Question 6 is 6 marks)



- 7 A cylinder is placed on the ground.

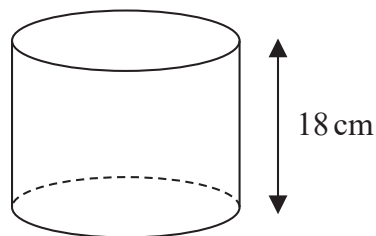


Diagram **NOT**  
accurately drawn

The height of the cylinder is 18 cm.

The force exerted by the cylinder on the ground is 72 newtons.

The pressure on the ground due to the cylinder is 1.4 newtons/cm<sup>2</sup>

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cylinder.

Give your answer correct to 3 significant figures.

$$P = 1.4 \text{ N/cm}^2$$

$$F = 72 \text{ N}$$

$$1.4 = \frac{72}{\text{Area}}$$

$$\text{Area} = \frac{72}{1.4} = 51.428\dots$$

$$\text{Volume} = \text{cross sectional area} \times \text{height}$$

$$= 51.428 \times 18$$

$$= 925.714\dots$$

$$\text{3.s.f.} \Rightarrow 926$$

$$\dots\dots\dots 926 \dots\dots\dots \text{cm}^3$$

(Total for Question 7 is 4 marks)



8 (a) Write 0.000089 in standard form.

$$8.9 \times 10^{-5}$$

(1)

(b) Write  $8.34 \times 10^4$  as an ordinary number.

$$83400$$

$$83400$$

(1)

(Total for Question 8 is 2 marks)

9 (a) Simplify  $8 \times (4t)^0$

$$(4t)^0 = 1 \quad \text{so } 8 \times 1 = 8$$

$$8$$

(1)

$$x^6 \div x^{-5} = x^p$$

(b) Find the value of  $p$

$$x^{6-(-5)} = x^{6+5} = x^{11}$$

$$\text{so } p = 11$$

$$p = 11$$

(1)

(c) Simplify fully  $(2k^2m^4)^3$

$$2^3 k^{2 \times 3} m^{4 \times 3}$$

$$= 8k^6m^{12}$$

$$8k^6m^{12}$$

(2)

(Total for Question 9 is 4 marks)



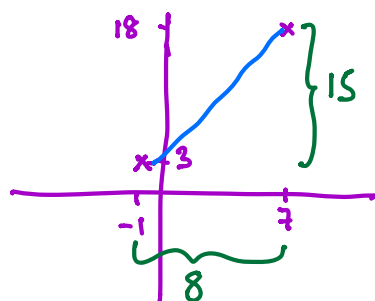


- 10 Two circles,  $C_1$  and  $C_2$ , are drawn on a centimetre grid, with a scale of 1 cm for 1 unit on each axis.

The centre of circle  $C_1$  is at the point with coordinates  $(-1, 3)$  and the radius of  $C_1$  is 13 cm.

The centre of circle  $C_2$  is at the point with coordinates  $(7, 18)$  and the radius of  $C_2$  is 6 cm.

- (a) Work out the distance between the centre of  $C_1$  and the centre of  $C_2$



$$\begin{aligned}
 \text{Distance } C_1 \rightarrow C_2 &= \sqrt{8^2 + 15^2} \\
 &= \sqrt{64 + 225} \\
 &= \sqrt{289} \\
 &= 17 \text{ units}
 \end{aligned}$$

..... 17 ..... cm  
(3)

- (b) Explain why circle  $C_1$  intersects circle  $C_2$

.....  
 $13 + 6 = 18$  and  $18 > 17$  so they must  
 intersect.  
 .....

(1)

(Total for Question 10 is 4 marks)



11 (a) Factorise  $9x^2 - 4y^2$

Difference of two squares

$$(3x - 2y)(3x + 2y)$$

$$(3x - 2y)(3x + 2y) \quad (2)$$

(b) Express  $\frac{7}{8} - \frac{x+3}{4x}$  as a single fraction in its simplest form.

$$\frac{7 \times 4x - 8(x+3)}{32x}$$

$$= \frac{28x - 8x - 24}{32x}$$

$$= \frac{20x - 24}{32x}$$

$$\div 4 = \frac{5x - 6}{8x}$$

$$\frac{5x - 6}{8x} \quad (3)$$

(Total for Question 11 is 5 marks)

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

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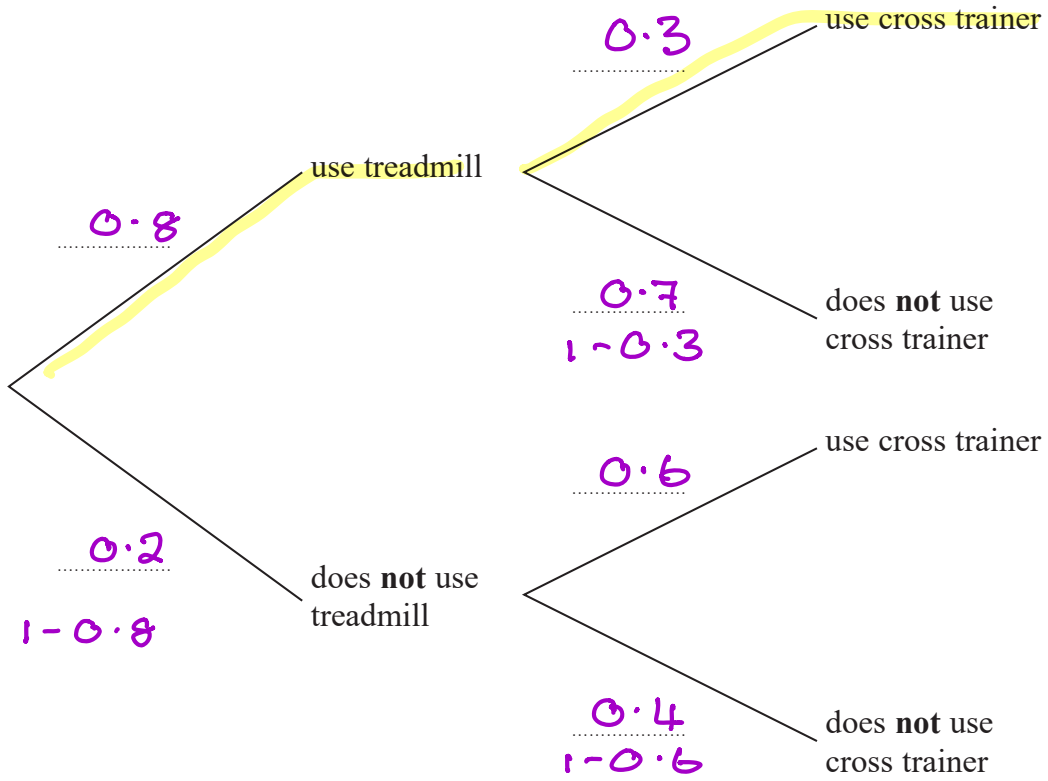
12 Rudolf goes to the gym.

The probability that he will use the treadmill is 0.8

When he uses the treadmill, the probability that he will use the cross trainer is 0.3

When he does **not** use the treadmill, the probability that he will use the cross trainer is 0.6

(a) Complete the probability tree diagram for this information.



(2)

(b) Work out the probability that Rudolf uses both the treadmill and the cross trainer.

$$0.8 \times 0.3 = 0.24$$

0.24

(2)

(Total for Question 12 is 4 marks)



- 13 Antoine is going on holiday.  
He makes 3 separate payments to cover the total cost of his holiday.

The following table shows how much money Antoine pays to the holiday company.

Payment	Amount paid
Payment 1	$\frac{3}{8}$ of the total cost
Payment 2	45% of the total cost
Payment 3	\$406

Work out how much Antoine has to pay for Payment 2

$$\frac{3}{8} + 45\% = 37.5\% + 45\%$$

$$= 82.5\%$$

$$100 - 82.5\% = 17.5\%$$

$$\begin{array}{l} \text{so } 17.5\% = 406 \\ \div 7 \downarrow 2.5\% = \\ \times 18 \downarrow 45\% = 1044 \end{array}$$

\$ 1044

(Total for Question 13 is 5 marks)



14 The function  $f$  is defined as

$$f: x \mapsto \frac{2x}{x-6} \quad x \neq 6$$

(a) Find  $f(10)$

$$\frac{2 \times 10}{10 - 6} = \frac{20}{4}$$

5

(1)

(b) Express the inverse function  $f^{-1}$  in the form  $f^{-1}: x \mapsto \dots$

$$y = \frac{2x}{x-6}$$

$$y(x-6) = 2x$$

$$yx - 6y = 2x$$

$$yx - 2x = 6y$$

$$x(y-2) = 6y$$

$$x = \frac{6y}{y-2}$$

$$\therefore f^{-1}(x) = \frac{6x}{x-2}$$

$$f^{-1}: x \mapsto \frac{6x}{x-2}$$

(3)

(Total for Question 14 is 4 marks)



- 15 Abraham is going to play a computer game.  
Abraham can win the game, draw the game or lose the game.

For any game that Abraham plays

the probability that he wins the game is 0.3  
the probability that he draws the game is 0.5  
the probability that he loses the game is 0.2

When Abraham wins a game, he scores +10 points.

When Abraham draws a game, he scores 0 points.

When Abraham loses a game, he scores -5 points.

Abraham plays 3 games and the points he scores in each of the 3 games are added together to get his total score.

Work out the probability that when he has played 3 games his total score is 0 points.

0 points => DDD  
WLL  
LLW  
LWL

$$\begin{aligned} &\Rightarrow 0.5^3 + 0.3 \times 0.2^2 + 0.2^2 \times 0.3 + 0.2 \times 0.3 \times 0.2 \\ &= 0.125 + 0.012 + 0.012 + 0.012 \\ &= 0.161 \end{aligned}$$

0.161

(Total for Question 15 is 4 marks)



16 Without using a calculator, show that  $\frac{12}{\sqrt{2}-1} - (\sqrt{2})^5 = 2\sqrt{32} + 12$

Show your working clearly.

$$\begin{aligned} \underline{\underline{\text{LHS}}} \quad \frac{12}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} &= \frac{12\sqrt{2}+12}{2+\sqrt{2}-\sqrt{2}-1} \\ &= \frac{12\sqrt{2}+12}{1} = 12\sqrt{2}+12 \end{aligned}$$

$$\begin{aligned} (\sqrt{2})^5 &= \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

$$\therefore 12\sqrt{2}+12 - 4\sqrt{2} = 8\sqrt{2}+12$$

$$\begin{aligned} \underline{\underline{\text{RHS}}} \quad \sqrt{32} &= \sqrt{16} \times \sqrt{2} = 4\sqrt{2} \\ 2\sqrt{32}+12 &= 2 \times 4\sqrt{2}+12 \\ &= 8\sqrt{2}+12 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

(Total for Question 16 is 3 marks)



- 17 A particle  $P$  moves along a straight line.  
The fixed point  $O$  lies on this line.

The displacement of  $P$  from  $O$  at time  $t$  seconds,  $t \geq 1$ , is  $s$  metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of  $P$  at time  $t$  seconds,  $t \geq 1$ , is  $v$  m/s

Work out the distance of  $P$  from  $O$  at the instant when  $v = 0$

$$\frac{ds}{dt} = 8t - \frac{125}{t^2}$$

$$0 = 8t - \frac{125}{t^2}$$

( $\times t^2$ )

$$0 = 8t^3 - 125$$

$$8t^3 = 125$$

$$t^3 = \frac{125}{8}$$

$$t = \sqrt[3]{\frac{125}{8}}$$

$$= \frac{5}{2} = 2.5 \text{ seconds}$$

$$\text{so } s = 4 \times 2.5^2 + \frac{125}{2.5}$$

$$= 25 + 50 = 75$$

75

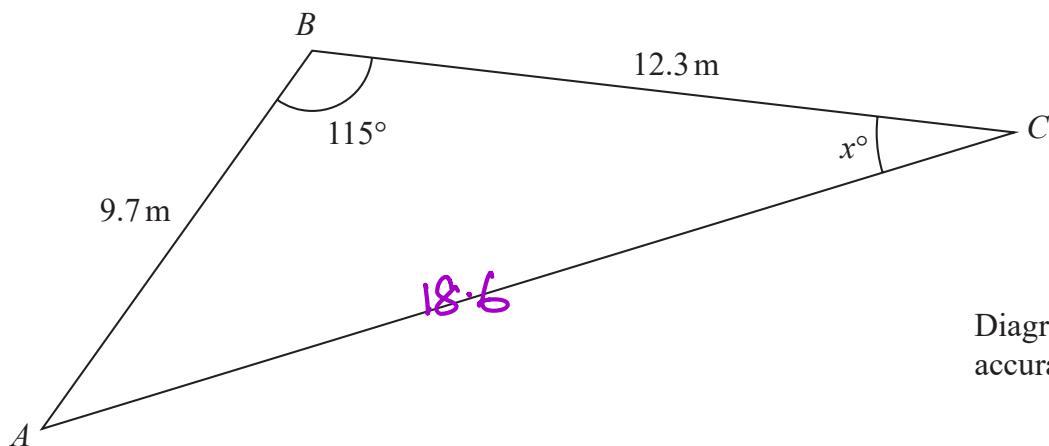
..... m

(Total for Question 17 is 5 marks)





18 Here is triangle  $ABC$



Work out the value of  $x$   
Give your answer correct to 3 significant figures.

$$AC^2 = 9.7^2 + 12.3^2 - 2 \times 9.7 \times 12.3 \times \cos 115$$
$$= 346.2 \dots$$

$$AC = \sqrt{346.2 \dots}$$
$$= 18.6 \dots$$

$$\frac{\sin x}{9.7} = \frac{\sin 115}{18.6 \dots}$$

$$\sin x = 9.7 \times \frac{\sin 115}{18.6 \dots}$$
$$= 0.47 \dots$$

$$x = \sin^{-1}(0.47 \dots)$$
$$= 28.2 \dots \text{ (3 s.f.)}$$

$$x = 28.2$$

(Total for Question 18 is 5 marks)



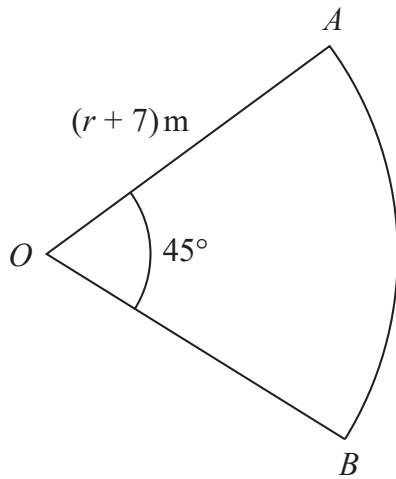


Diagram NOT  
accurately drawn

$OAB$  is a sector  $S$  of a circle with centre  $O$  and radius  $(r + 7)$  metres.  
Angle  $AOB = 45^\circ$

A circle  $C$  has radius  $(r - 2)$  metres.

The area of sector  $S$  is twice the area of circle  $C$

Find the value of  $r$

Show your working clearly.

sector S

$$\frac{45}{360} \times \pi \times (r+7)^2 = 2 \times \pi \times (r-2)^2$$

$$\frac{1}{8} \pi (r^2 + 14r + 49) = 2\pi (r^2 - 4r + 4)$$

$\times 8$

$$r^2 + 14r + 49 = 16(r^2 - 4r + 4)$$

$$= 16r^2 - 64r + 64$$

$$0 = 15r^2 - 78r - 15$$

$\div 3$

$$0 = 5r^2 - 26r - 5$$

$$(5r - 1)(r - 5) = 0$$

$$\therefore r = \frac{1}{5} \text{ or } r = 5$$

not valid

$$\therefore \underline{r = 5}$$

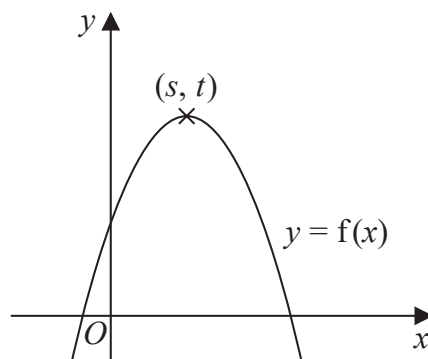


Question 19 continued.

$$r = \dots\dots\dots$$

(Total for Question 19 is 5 marks)

20 The diagram shows a sketch of part of the curve with equation  $y = f(x)$



There is one maximum point on this curve.

The coordinates of this maximum point are  $(s, t)$

Find, in terms of  $s$  and  $t$ , the coordinates of the maximum point on the curve with equation

(i)  $y = f(x - 2)$

$\rightarrow 2$

$s+2, t$

$(s+2, t)$   
(1)

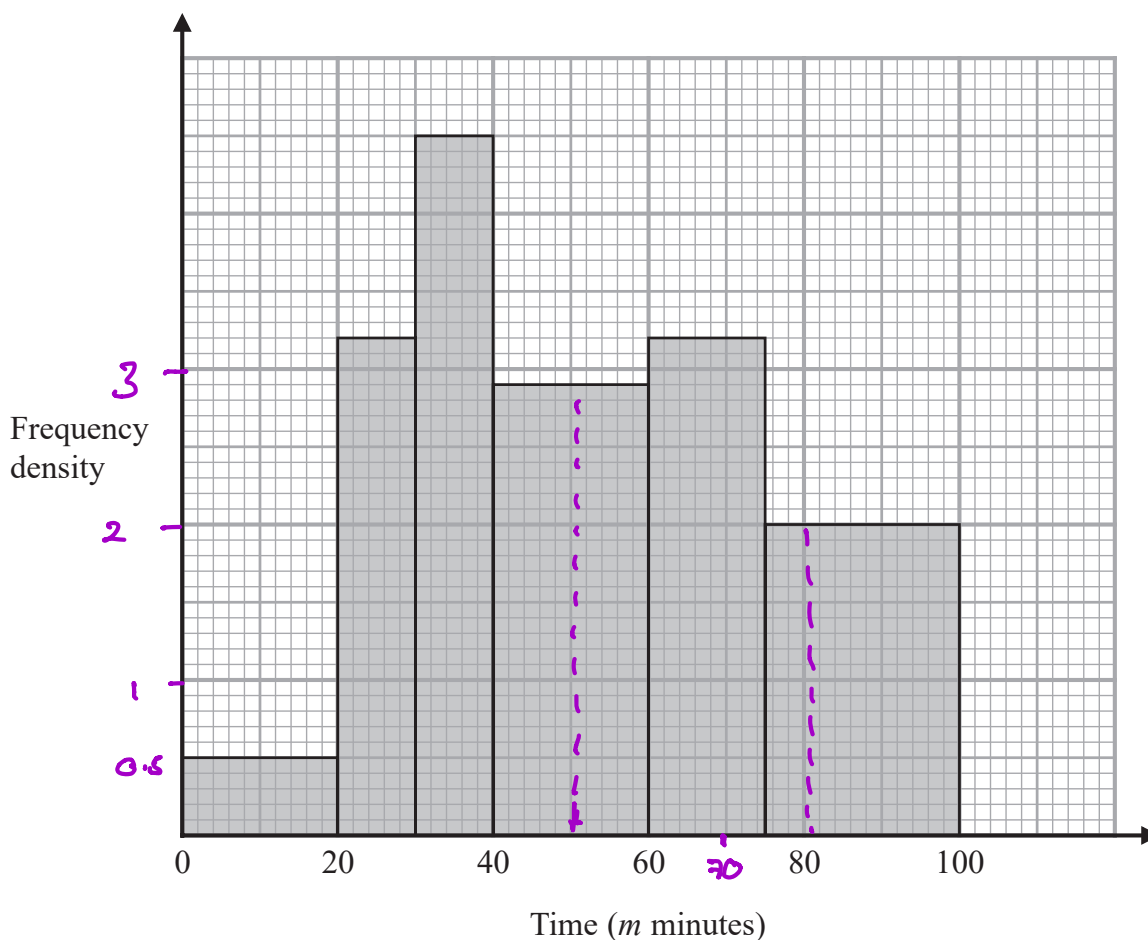
(ii)  $y = 3f(x)$

$(s, 3t)$   
(1)

(Total for Question 20 is 2 marks)



- 21 The histogram shows information about the total time,  $m$  minutes, taken by each child in a school to walk to school every day for one week.



There are no children for whom  $m > 100$

There are 10 children for whom  $m \leq 20$

Work out an estimate for the number of children for whom  $50 < m \leq 80$

	Frequency	F.D
0 - 20	10	$\frac{10}{20} = 0.5$

50 → 80

$$50 \rightarrow 60 = 2.9 \times 10 = 29$$

$$60 \rightarrow 75 = 3.2 \times 15 = 48$$

$$75 \rightarrow 80 = 2 \times 5 = 10$$

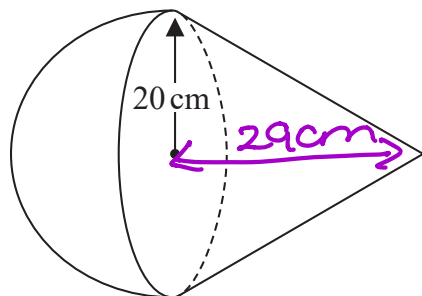
$$\underline{86}$$

86

(Total for Question 21 is 3 marks)



22 A solid is made from a cone and a hemisphere.



$$SA = \pi r l$$

$$V = \frac{1}{3} \pi r^2 h$$

Diagram NOT accurately drawn

$$SA = 4\pi r^2$$

The circular plane face of the hemisphere coincides with the circular base of the cone. The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is  $580\pi \text{ cm}^2$

The volume of the solid is  $k\pi \text{ cm}^3$

Work out the exact value of  $k$

Cone.

$$\pi \times r \times l = 580\pi$$

$$l = \frac{580\pi}{20\pi} = 29$$

$$20 \quad \begin{array}{c} \triangle \\ l=29 \\ h=? \end{array} \quad h = \sqrt{29^2 - 20^2} = \sqrt{441} = 21$$

SOLID

$$\frac{1}{2} \times \frac{4^2}{3} \times \pi \times 20^3 + \frac{1}{3} \pi \times 20^2 \times 21 = k\pi$$

$$\frac{16000}{3} + 2800 = k$$

$$k = \frac{24400}{3}$$

$$k = \frac{24400}{3}$$

(Total for Question 22 is 5 marks)



23 A polygon has  $n$  sides, where  $n > 5$

When arranged in order of size, starting with the largest number, the sizes of the interior angles of the polygon, in degrees, are the terms of an arithmetic sequence.

Here are the first five terms of this sequence.

$$177 \quad 175 \quad 173 \quad 171 \quad 169$$

$$\quad \quad \quad \underline{-2} \quad \underline{-2} \quad \underline{-2} \quad \underline{-2}$$

Find the value of  $n$

Show clear algebraic working.

$$\text{Common difference} = -2$$

$$\begin{aligned} S_n &= \frac{n}{2} (2 \times 177 + (n-1) \times -2) \\ &= \frac{n}{2} (354 - 2n + 2) \\ &= \frac{n}{2} (356 - 2n) \\ &= n(178 - n) \end{aligned}$$

so

$$178n - n^2 = (n-2)180$$

$$0 = n^2 - 178n + 180n - 360$$

$$= n^2 + 2n - 360$$

$$0 = (n + 20)(n - 18)$$

$$\therefore n = 18 \quad n = -20$$

not valid



Question 23 continued

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

$$n = \dots\dots\dots 18 \dots\dots\dots$$

(Total for Question 23 is 6 marks)

Turn over for Question 24



P 6 8 7 9 6 A 0 2 5 2 8

24 Express each of  $a$ ,  $b$  and  $c$  in terms of  $q$  so that

$$q + 12x - qx^2$$

can be written as  $a - b(x - c)^2$

$$-qx^2 + 12x + q$$

$$\Rightarrow -q \left( x^2 - \frac{12x}{q} - 1 \right)$$

$$\Rightarrow -q \left[ \left( x - \frac{6}{q} \right)^2 - \frac{36}{q^2} - 1 \right]$$

$$\Rightarrow -q \left( x - \frac{6}{q} \right)^2 + \frac{36}{q} + q$$

$$= \underbrace{\frac{36}{q} + q}_a - \underbrace{q}_b \left( x - \underbrace{\frac{6}{q}}_c \right)^2$$

$$a = \frac{36}{q} + q$$

$$b = q$$

$$c = \frac{6}{q}$$

(Total for Question 24 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

